SEMESTER-II

PERIOD-IV

M12CH12

TOPIC

12 Numbers and Numeration

12.1. VARIOUS TYPES OF NUMBERS

1. **Natural numbers:** *Counting numbers are called natural numbers.*
   Thus 1, 2, 3, 4, ... are all natural numbers.
2. **Whole numbers:** *All Counting numbers, together with 0; from the set of whole numbers.*
   Thus 1, 2, 3, 4, ... are all whole numbers.
3. **Integers:** *All Counting numbers zero and negatives of counting numbers, form the set of integers.*
   Thus, ... – 3, – 2, – 1, 0, 1, 2, 3 .... are all integers.
   Set of positive integers = {1, 2, 3, 4, 5, 6, ...}
   Set of negative integers = {– 1, – 2, – 3, – 4, – 5, – 6, ...}
   Set of all non-negative integers = {0, 1, 2, 3, 4, 5, ...}
4. **Even numbers:** *A counting number divisible by 2 is called an even number.*
   Thus, 0, 2, 4, 6, 8, 10, 12, ... etc. are all even numbers.
5. **Odd numbers:** *A counting number not divisible by 2 is called an odd number.*
   Thus, 1, 3, 5, 7, 9, 11, 13, ... etc. are all odd numbers.
6. **Prime numbers:** *A counting number is called a prime numbers if it has exactly two factors, namely itself and 1.*
   Ex. All prime numbers less than 100 are
   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

7. **Composite numbers:** *All counting numbers, which are not prime, are called composite numbers.*

   A composite numbers has more than 2 factors.

8. **Perfect numbers:** *A number, the sum of whose factors (except the number itself) is equal to the number is called a perfect numbers, e.g. 6, 28, 496.*

   The factors of 6 are 1 2, 3, and 6 And, 1 + 2 + 3 = 6.

   The factors of 28 are 1 2, 4, 7, 14 and 28 And, 1 + 2 + 4 + 7 + 14 = 28.

9. **Co-primes (or Relative numbers):** *Two numbers whose H.C.F is 1 are called co-primes numbers,*

   Ex. (2, 3), (8, 9) are pairs of co-primes.

10. **Twin primes:** *Two prime numbers whose difference is 2 are called twin-primes.*

    Ex. (3, 5), (5, 7), (11, 13) are pairs of twin-primes.

11. **Rational numbers:** *Numbers which can be expressed in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$ are called rational numbers.*

    Ex. $\frac{1}{8}, \frac{-8}{11}, 0, 6, 5\frac{2}{3}$ etc.

12. **Irrational numbers:** *Numbers which can be expressed in decimal would be in non-terminating and non-repeating form, are called irrational numbers.*

    Ex. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi, e,$ 0.231764735.......

Important Facts

1. All natural numbers are whole numbers.
2. All whole numbers are not natural numbers.
3. Even number + Even number = Even number

   Odd number + Odd number = Even number

   Even number + Odd number = Odd number

   Even number – Even number = Even number

   Odd number – Odd number = Even number

Even number – Odd number = Odd number

Odd number – Even number = Odd number

Even number × Even number = Even number

Odd number × Odd number = Even number

Even number × Odd number = Even number

4. The smallest prime number is 2.
5. The only even prime number is 2.
6. The first odd prime number is 3.
7. 1 is a unique number – neither prime not composite.
8. The least composite number is 4.
9. The least odd composite number is 9.
10. Test for a number to be prime.

    Let p be a given number and let n be the smallest counting numbers such that $n^2 \geq p$.

    Now, test whether p is divisible by any of the prime numbers less than or equation to n. If yes, then p is not prime otherwise, p is prime.

Example 1. *Test, which of the following are prime numbers*?

(i) 137 (*ii*) 173 (*iii*) 319

(*iv*) 437 (*v*) 811

Solution. (*i*) We know that $(12)^2 > 137$

Prime numbers less than 12 are 2, 3, 5, 7, 11

Clearly, none of them divides 137.

∴ 137 is a prime number.

(*ii*) We know that $(14)^2 > 173$

Prime numbers less than 14 are 2, 3, 5, 7, 11, 13

Clearly, none of them divides 173.

∴ 173 is a prime number.

(*iii*) We know that $(18)^2 > 319$

Prime numbers less than 18 are 2, 3, 5, 7, 11, 13, 17

Out of these prime numbers, 11 divides 319 completely.

∴ 319 is not a prime number.

(*iv*) We know that $(21)^2 > 437$

Prime numbers less than 21 are 2, 3, 5, 7, 11, 13, 17, 19

Clearly, 437 is divisible by 19.

$\therefore$ 437 is not a prime number.

(*v*) We know that $(30)^2 > 811$

Prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Clearly, none of these numbers divides 811.

$\therefore$ 811 is a prime number.

Important Formulae

(*i*) $(a + b)^2 = a^2 + b^2 + 2ab$

(*ii*) $(a - b)^2 = a^2 + b^2 - 2ab$

(*iii*) $(a + b)^2 (a - b)^2 = 2(a^2 + b^2)$

(*iv*) $(a + b)^2 (a - b)^2 = 4ab$

(*v*) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(*vi*) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(*vii*) $a^2 - b^2 = (a + b)(a - b)$

(*viii*) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(*ix*) $a^3 + b^3 = (a + b)(a^2 + b^2 - 2ab)$

(*x*) $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

(*xi*) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(*xii*) If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

1. **Divisibility by 2:** *A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.*

   Ex. 58694 is divisible by 2, while 86945 is not divisible by 2.

2. **Divisibility by 3:** *A number is divisible by 3 only when the sum of its digits is divisible by 3.*

   Ex.

   (*i*) In the number 695421 the sum of digits = 27, which is divisible by 3.

   $\therefore$ 695421 is divisible by 3

   (*ii*) In the number 948653, the sum of digits = 35, which is not divisible by 3.

   $\therefore$ 948653, is not divisible by 3

3. **Divisibility by 9:** *A number is divisible by 9 only when the sum of its digits is divisible by 9.*

Ex.

(*i*) In the number 246591, the sum of digits = 27, which is divisible by 9.

∴ 246591 is divisible by 9

(*ii*) In the number 734519, the sum of digits = 29, which is not divisible by 9.

∴ 734519 is not divisible by 9.

4. **Divisibility by 4:** *A number is divisible by 4 if the number formed by its last two digits is divisible by 4.*

Ex.

(*i*) 6879376 is divisible by 4, since 76 is divisible by 4.

(*ii*) 496138 is not divisible by 4, since 38 is not divisible by 4.

5. **Divisibility by 8:** *A number is divisible by 8 if the number formed by its last three digits is divisible by 8.*

Ex.

(*i*) In the number 16789352, the number formed by last 3 digits, namely 325 is divisible by 8,

∴ 16789352 is divisible by 8.

(*ii*) In the number 576484, the number formed by last 3 digits, namely 484 is not divisible by 8,

∴ 576484 is not divisible by 8.

6. **Divisibility by 10:** *A number is divisible by 10 only when its unit digit is 0.*

Ex.

(*i*) 7849320 is divisible by 10, since its unit digit is 0.

(*ii*) 678405 is not divisible by 10, since its unit digit is not 0.

7. **Divisibility by 5:** *A number is divisible by 5 only when its unit digit is 0 or 5.*

Ex.

(*i*) Each of the numbers 76895 abd 68790 is divisible by 5.

8. **Divisibility by 11:** *A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a number divisible by 11.*

Ex.

(*i*) Consider the number 29435417

(Sum of its digits at odd places) – (Sum of its digits at even places) = (7 + 4 + 3 + 9) – (1 + 5 + 4 + 2) = (23 – 12) = 11, which is divisible by 11.

∴ 29435417 is divisible by 11.

(*ii*) Consider the number 57463822

(Sum of its digits at odd places) – (Sum of its digits at even places) = (2 + 8 + 6 + 7) – (2 + 3 + 4 + 5) = (23 – 14) = 9, which is not divisible by 11.

∴ 57463822 is not divisible by 11.

9. Divisibility by 25: *A number is divisible by 25 if the number formed by its last two digits is either 00 or divisible by 25.*

Ex.

(*i*) In the number 63875, the number formed by last 2 digits, namely 75 is divisible by 25.

∴ 63875 is divisible by 25.

(*ii*) In the number 96445, the number formed by last 2 digits, namely 45 is divisible by 25.

∴ 96445 is not divisible by 25.

10. Divisibility by 7 or 13: *Divide the number into groups of 3 digit: (starting from right) and find the difference between the sum of the numbers in odd and even places. If the difference is 0 or divisible by 7 or 13 (as the case may be), it is divisible by 7 or 13.*

Ex.

(*i*) 4537792 → 4/537/792

(792 + 4) – 537 = 259, which is divisible by 7 but not by 13.

∴ 4537792 is divisible by 7 and not by 13.

(*ii*) 579488 → 579/488

579 – 488 = 91, which is divisible by both 7 and 13.

∴ 579488 is divisible by both 7 and 13.

11. Divisibility by 16: *A number is divisible by 16, if the number formed by its last 4 digits is divisible by 16.*

Ex.

(*i*) In the number 463776, the number formed by last 4 digits, namely 3776, is divisible by 16.

∴ 463776 is divisible by 16.

(*ii*) In the number 895684, the number formed by last 4 digits, namely 5684, is not divisible by 16.

∴ 895684 is not divisible by 16.

12. Divisibility by 6: *A number is divisible by 6, if it is divisible by both 2 and 3.*

13. Divisibility by 12: *A number is divisible by 12, if it is divisible by both 3 and 4.*

14. Divisibility by 15: *A number is divisible by 15, if it is divisible by both 3 and 5.*

15. Divisibility by 18: *A number is divisible by 18, if it is divisible by both 2 and 9.*

16. Divisibility by 14: *A number is divisible by 14, if it is divisible by both 2 and 7.*

17. Divisibility by 24: *A number is divisible by 24, if it is divisible by both 3 and 8.*

18. Divisibility by 40: *A number is divisible by 40, if it is divisible by both 5 and 8.*

19. Divisibility by 80: *A number is divisible by 80, if it is divisible by both 5 and 16.*

Note: *If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.*

If p and q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

Ex. 36 is divisible by both 4 and 6, but it is not divisible by (4 × 6) = 24, since 4 and 6 are not co-primes.

12.2. FACTORIAL OF A NUMBER

Let *n* be a positive integer.

Then, the continued product of first n natural numbers is called factorial n, denoted by n! or $\lfloor\underline{n}$

Thus, $n! = n(n-1)(n-2) \ldots 3.2.1$

Ex. 5! = 5 × 4 × 3 × 2 × 1 = 120

Note: 0! = 1

12.3. MODULUS OF A NUMBER

$$|x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$

Ex. | – 5 | = 5, |4| = 4, |– 1| = 1, etc.

12.4. GREATEST INTEGRAL VALUE

The greatest integral value of an integer x, denoted by [x], is defined as the greatest integer not exceeding x.

Ex. [1.35] = 1, $\left[\frac{11}{4}\right] = \left[2\frac{3}{4}\right] = 2$, etc.

12.5. MULTIPLICATION BY SHORT CUT METHODS

1. **Multiplication by Distributive Law:**

   (*i*) $a \times (b + c) = a\,b + a\,c$

   (*ii*) $a \times (b - c) = a\,b - a\,c$

   Ex.

   (*i*) 567958 × 99999

   = 567958 × (100000 – 1) = 567958 × 100000 – 567958 × 1

   = (59795800000 – 597958) = 56795232042

   (*ii*) 978 × 184 + 978 × 816

   = 978 × (184 + 816) = 978 × 1000 = 978000

2. **Multiplication of a Number by 5:** Put n zeros to the right of the multiplicand and divide the number so formed by 2^n.

   Ex. 975436 × 625 = 975436 × 5^4 = $\frac{9754360000}{16}$ = 609647500

12.6. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM

If we divide a given number by another number then

Dividend = (Divisor × Quotient) + Remainder

Important Facts

1. (*i*) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

(*ii*) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.

(*iii*) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

2. To find the highest power of a prime number p in $n!$

Highest power of p in $n! = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots + \left[\frac{n}{p^r}\right]$, where

$p^r \le n < p^{r+1}$

Example 2. *Simplify:*

(*i*) $\dfrac{789 \times 789 \times 789 + 211 \times 211 \times 211}{789 \times 789 - 789 \times 211 + 211 \times 211}$

(*ii*) $\dfrac{658 \times 658 \times 658 - 328 \times 328 \times 328}{658 \times 658 + 658 \times 328 + 328 \times 328}$

Solution. (*i*) Given exp. $= \dfrac{(789)^3 + (211)^3}{(789)^2 - (789 \times 211) + (211)^2} = \dfrac{a^3 + b^3}{a^2 - ab + b^2}$

(where $a = 789$ and $b = 211$)

(*ii*) Given exp. $= \dfrac{(658)^3 - (328)^3}{(658)^2 + (658 \times 328) + (328)^2} = \dfrac{a^3 - b^3}{a^2 + ab + b^2}$

(where $a = 658$ and $b = 328$)

Important Facts and Formulae

I. Square Root: If $x^2 = y$ we say that the square root of y, is x and we write, $\sqrt{y} = x$.

II. Cube Root: The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$

Thus, $\sqrt[3]{x} = \sqrt[3]{2 \times 2 \times 2} = 2, \sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.

Note: 1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$ 2. $\sqrt{\frac{x}{y}} = \sqrt{\frac{x}{y}} = \sqrt{\frac{x}{y}} \times \sqrt{\frac{y}{y}} = \sqrt{\frac{xy}{y}}$

Example 3. *Evaluate* $\sqrt{6084}$ *by factorization method:*

Solution. Method; Express the given number as the product of prime factors, Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.

2	6084
2	3042
3	1521
3	507
13	169
	13

Thus, resolving 6084 into prime factors, we get

$$6084 = 2^2 \times 3^2 \times 13^2$$

$$\therefore \quad \sqrt{6084} = (2 \times 3 \times 13) = 78.$$

Important Facts and Formulae

I. Logarithm: If a is a positive real number, other than 1 and $a^m = x$, then we write: $m = \log_a x$ and we say that the value of log x to the base a is m.

Example:

(*i*) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

(*ii*) $3^4 = 81 \Rightarrow \log_3 81 = 4$

(*iii*) $2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$

(*iv*) $(.1)^2 = 0.1 \Rightarrow \log_{(.1)} 01 = 2$

II. Properties of Logarithms:

1. $\log_a (xy) = \log_a x + \log_a y$
2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_x x = 1$
4. $\log_a 1 = 0$
5. $\log_a (x^p) = p(\log_a x)$
6. $\log_a x = \frac{1}{\log_x a}$
7. $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a}$
8. $a^{\log_a x} = x$
9. $x^{\log_a y} = y^{\log_a x}$
10. $\log_a x^p = \frac{p}{q} \log_a x$

Remember: When base is not mentioned, it is taken as 10.

III. Common Logarithms: Logarithms to the base 10 are known as common logarithms.

IV. The logarithm of a number contains two parts, namely *characteristic* and *mantissa*.

Characteristic: The integral part of the logarithm of a number is called its *characteristic*.

Case I: When the number is greater than 1.

In this case, the characteristic is one less than the number of digits to the left of the decimal point in the given number.

Case II: When the number is less than 1.

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of – 1, – 2, etc, we write $\bar{1}$ (one bar), $\bar{2}$ (two bar) etc.

Example:

Number	*Characteristic*	*Number*	*Characteristic*
348.25	2	0.6173	$\bar{1}$
46.583	1	0.03125	$\bar{2}$
9.2193	0	0.00125	$\bar{3}$

Mantissa: The decimal part of the logarithm of a number is known as it *mantissa*. For mantissa, we look through log table.

Example 4. *Evaluate:*

(*i*) $\log_3 27$ (*ii*) $\log_7\left(\frac{1}{343}\right)$

(*iii*) $\log_{100}(0.01)$ (*iv*) $\log_8 128$

Solution. (*i*) $\log_3 27 = \log_3 3^3 = 3 \log 3^3 = 3$.

(*ii*) $\log_7\left(\frac{1}{343}\right) = \log_7\left(\frac{1}{7^3}\right) = \log_7 7^{-3} = -3 \log_7 7 = -3$.

(*iii*) Let $\log_{100}(0.01) = \log_{100}\left(\frac{1}{100}\right)$

$= \log_{100}(100)^{-1} = -1 \log_{100} 100 = -1$

(*iv*) Let $\log_8 128 = \log_{2^3}(2^7) = \frac{7}{3}\log_2 2 = \frac{7}{3}$

Important Facts and Formulae

I. If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$

II. If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days

III. If A is thrice as good a workman as B, then
Ratio of work done by A and B = 3 : 1
Ratio of times taken by A and B to finish a work = 1 : 3

Example 5. *If Roger can do a piece of work in 8 days and Antony can complete the same work in 5 days, in how many days will both of them together complete it?*

Solution. Roger's 1 day's work = $\frac{1}{8}$; Antony's 1 day's work = $\frac{1}{5}$.

(Roger + Antony)'s 1 day's work = $\left(\frac{1}{8}+\frac{1}{5}\right) = \frac{13}{40}$

∴ Both Roger and Antony will complete the work in $\frac{40}{13} = 3\frac{1}{3}$ days.

12.7. NUMBER OF BINARY OPERATIONS

Let S be a finite set consisting of n elements. Then, S × S has n^2 elements. Since a binary operation on S is a function from S × S to S. Therefore, the total number of binary operations on S is equation to the number of functions from S × S to S. We know that the total number of functions from a finite set A to a finite set B is $(n(B))^{n(A)}$. Therefore, the total number of binary operations on S is n^{n^2}.

For example, if S = (a, b) then $2^{2^2} = 2^4 = 16$ binary operations can be defined on S.

Remark: If '*' is a binary operation on a set S, then we also say that 'S' is closed with respect to '*'.

Clearly, the set E of all even integers is close with respect to addition but the set O of odd integers is not closed with respect to addition as $1 \in O$, $5 \in O$ but $1 + 5 \notin O$.

12.8. TYPES OF BINARY OPERATIONS

Consider a binary operation '*' on a set S. For any two distinct elements in S, we have

$$(a, b) \neq (b, a)$$

Since '*': $S \times S \rightarrow S$. Therefore, * (a, b) and * (b, a) *i.e.,* images of (a, b) and (b, a) under '*' may or may not be same. In other words, $a * b$ and $b * a$ may or may not be equal. Thus, it is not necessary that for a binary operation * on a set S, $a * b = b * a$ must hold for all $a, b \in S$. If $a * b = b * a$ for all $a, b \in S$, then we say that binary operation * possesses commutativity as defined below.

COMMUTATIVITY: *A binary operation '*' on a set S is said to be a commutative binary operation if $a * b = b * a$ for all $a, b \in S$.*

The binary operations addition (+) and multiplication (×) are commutative binary operations on Z. However, the binary operation subtraction (–) is not a commutative binary operation on Z as 3 – 2 = 2 – 3.

Clearly, $\frac{ab}{2} = \frac{ba}{2}$ for all $a, b \in Q - \{0\}$

[$\because$ Multiplication is commutative on Q – {0}]

$\therefore$ $a * b = b * a$ for all $a, b \in Q - \{0\}$

So, * is commutative on Q – {0}.

Illustration 2. *Let * be a binary operation on R, the set of all real numbers, defined by $a * b = \sqrt{a^2 + b^2}$ for all $a, b \in R$. Show that * is commutative.*

Solution. We have

$$\therefore \qquad a * b = \sqrt{a^2 + b^2} \text{ and } b * a = \sqrt{b^2 + a^2}$$

for all $a, b \in$ Q – {0}

$$\text{But,} \qquad \sqrt{a^2 + b^2} = \sqrt{b^2 + a^2} \text{ for all } a, b \in \text{R}$$

$$\Rightarrow \qquad a * b = b * a \text{ for all } a, b \in \text{R}$$

So, * is commutative on R.

ASSOCIATIVITY: *A binary operation '*' on a set S is said to be an associative binary operation, if (a* b) * c = a * (b * c)* for all $a, b \in$ S.

The binary operations of addition (+) and multiplication (×) are associative binary operation on Z. However, the binary operation subtraction (–) is not as associative binary operation on Z as (2 – 3) – 5 ≠ 2 – (3 – 5).

If S is a non-empty set, then union (∪) and intersection (∩) are both commutative and associative binary operation on P(S) the power set of set (S) as

$$A \cup B = B \cup A,\ A \cap B = B \cup A$$

$$(A \cup B) \cup C = A \cup (B \cup C) \text{ and } (A \cap B) \cap C$$

$$= A \cap (B \cap C) \text{ for all } A, B, C \in P(S).$$

Illustration 3. *Addition of vectors is commutative as well as associative on the set V_3 of all vectors in 3-dimensional space. However, "cross-product" is neither commutative not associative on V_3.*

Illustration 4. *Addition of matrices is commutative as well as associative binary operation on $R^{m\times n}$ (set of all m × n matrices over R). Multiplication of matrices is not commutative but it is associative on $R^{n\times n}$ (set of all square m × n matrices over R). Multiplication of matrices is not commutative but it is associative on $R^{m\times n}$ (set of all square matrices of order n over R).*

Illustration 5. *Let S denote the set of all functions from a non-empty set A to itself. Clearly, composition of functions 'o' is a binary operation on S such that*

$$fog \neq gof \text{ but } (fog)oh = fo(goh) \text{ for all } f, g, h \in S.$$

Hence, composition of functions 'o' is associative but not a commutative binary operation on S.

Illustration 6. *If the operation * is defined on the set Q of all rational numbers by the rule* $a * b = \frac{ab}{3}$ *for all a, b* ∈ *Q. Show that * is associative on Q.*

Solution. Let $a, b, c \in$ Q *Then*

$$(a * b) * c = \frac{ab}{3} * c = \frac{\left(\frac{ab}{3}\right)c}{3} = \frac{(ab)c}{9} \qquad \ldots(i)$$

and
$$a * (b * c) = \frac{a * \left(\frac{bc}{3}\right)}{3} = \frac{a\left(\frac{bc}{3}\right)}{3} = \frac{a(bc)}{9} \qquad \ldots(ii)$$

Since multiplication is associative on Q.

$$\therefore \qquad (ab)c = a(bc)$$

$$\Rightarrow \qquad \frac{(ab)c}{9} = \frac{a(bc)}{9}$$

$$\Rightarrow \qquad (a * b) * c = a * b * c \qquad \text{[By using } (i) \text{ and } (ii)\text{]}$$

Thus, $(a * b) * c = a * (b * c)$ for all $a, b, c \in$ Q. Hence, * is associative on Q.

DISTRIBUTIVITY: Let S be a non-empty set and * and '⊙' be two binary operations on S. Then '*' is said to be distributive over ⊙, if for all $a, b, c \in$ S.

$$a * (b \odot c) = (a * b) \odot (a * c) \qquad \text{[Left distributivity of * over } \odot\text{]}$$

and
$$(b \odot c) * a = (b * a) \odot (c * a) \qquad \text{[Right distributivity of * over } \odot\text{]}$$

The binary operation multiplication (·) on Z is distributive over the binary operation addition (+) on Z because

$$a \cdot (b + c) = a.b + a.c$$

and
$$(b + c).a = b.a + c.a \text{ for all } a, b, c \in \text{Z}.$$

However, addition (+) is not distributive over multiplication (·) because

$$2 + (3 \times 5) \neq (2 + 3) \times (2 + 5)$$

If S is a non-empty set, then union (∪) is distributive over intersection (∩) on P(S), because

$$\text{A} \cup (\text{B} \cap \text{C}) = (\text{A} \cup \text{B}) (\text{A} \cup \text{C}) \text{ for all A, B, C} \in \text{P(S)}.$$

Also, intersection (∩) is distributive over union (∪) on P(S).

EXERCISE

1. Evaluate:

(*i*) $\log_7 1 = 0$ (*ii*) $\log_{34} 34$ (*iii*) $36^{\log 64}$ (*iv*) $\log_8 128$

2. A and B together can complete a piece of work in 15 days and B alone in 20 days, In how many day can A alone complete the work?

7. Examine whether the binary operation * defined on R by $a * b = ab + 1$ is associative or not.

4. '*' is a binary operation defined on R, the set of all real numbers, by $a * b = \sqrt{a^2 + b^2}$ for all $a, b \in$ R. Show that * is associative on R.